PRACTICAL 3: GR4J

# River discharge response to rainfall changes

SETTING UP

1. Create a folder for the practical in kname\My Documents or Desktop.
2. Open the GR4J model Excel file from KEATS.
3. Excel models require you to change a setting to calculate equations:  
     
   **File - Options - Formulas   
   Select the tick box on the right**  
     
   **Remote users on Macs:**  
   Excel - Preferences.  
   Tick **Use iterative calculation**
4. Now, whenever you want to run a simulation, press:  
     
   Windows: F9 key.   
     
   **Remote users on Macs**: 'Command' key together with '=', i.e. <COMMAND>+<=>
5. Note this applies to ALL calculations, so copying and pasting a formula into a new cell (e.g. =SUM() or =AVERAGE()) will ***not*** calculate the answer until you press the key(s) above.

1. GETTING TO KNOW THE MODEL

First, we will explore the model in detail, to see what it is capable of and to check we can run a new simulation.

# 1.1 Exploring GR4J

We will again go through each of the following questions from "Get to know your model" in Lecture 2:

1. Does it run? And can you get the numerical outputs?
2. What are the state variable(s)? Which one(s) will you focus on?
3. What is the domain? e.g. region, point location, abstract
4. Can you change the forcings (a.k.a. boundary conditions)?
5. Do you have the data/information for the past and your science question(s)?
6. Do you have the initial conditions?
7. Can you change the parameters?

**a) Does it run? And can you get the numerical output?**

1. Make a note of the **Bias** value (E35) - this will be a quick way to check if the model output has changed.
2. Change the value of one of the parameters **x1-x4** and run the model using the key(s) given above.  
     
   Check if the Bias value has changed. If so, you have successfully run the model!
3. Undo the edit, so you **go back to the default parameter value**, and **run the model** again. (This is important for the calculations below!)
4. The **simulated streamflow** (**Q**) values are calculated in the column AZ, and plotted in the worksheet labelled Q (red line).  
     
   You have successfully accessed the numerical output!
5. The graph also shows the **observed streamflow** values (blue line; data in column E) and input **precipitation** (black line; data in column B).

**b) What are the state variables? Which ones will you focus on?   
c) What is the domain?**

1. For (b), you looked at this in the preparation quiz.   
     
   The key state variables are:   
     
   the **amount of water in the two stores** (S and R, expressed as fractions of the maximum values, x1 and x3), which are calculated in columns N and AX and plotted in the worksheets S and R;   
     
   the **streamflow** (Q, discussed in the previous question).  
     
   We will focus on Q as the most important output.
2. For (c), the domain is the catchment you looked at in the preparation quiz, **Le Léguer à Belle-Isle-en-Terre** (with an area of 260 km2).

**d) Can you change the forcings (a.k.a. boundary conditions)?**

1. As you saw in the preparation, the forcings (**precipitation** and **potential evapotranspiration**, P and PET) are provided in columns B and C.   
     
   So yes, it is easy to change these values.
2. However, the model is currently only set-up to use forcings for the past, observed over the same time period as the calibration data.  
     
   To use scenarios of plausible future forcings under climate change, we will have to modify it a bit.   
     
   We will only change the **precipitation** forcing in this practical.  
     
   In preparation for this, add a new worksheet, called **Forcing.**
3. Copy and paste the precipitation forcing into it, including the header cell ("Precip (mm)").   
     
   Add another header above this for the date, e.g.: "Present day" or "1991-92" (still feels fairly recent to some of us...).

**e) Do you have the data/information for the past and your science question(s)?**

**f) Do you have the initial conditions?**

1. **Past:** As you have found, GR4J is already set up to simulate past streamflow for 1991-1992, i.e. it already has the forcing data for this.   
     
   For our science question, we will use an **idealised** scenario of climate change by **doubling the 1991-92 precipitation values**.  
     
   In the Forcing worksheet, create a new column of 2x the precipitation values. Add a header above this, e.g. "Future scenario" or "Doubled".
2. **Initial conditions:** as you saw in the preparation, we have some initial values for the water stores in this catchment (E16 and E17), and will assume these are suitable for any simulation.

**g) Can you change the parameters?**

1. Yes - we have already tried this. The values are in cells D10-D13.
2. Let's look at these in a little more detail to understand them fully....   
     
   The **actual values of x1-x4** used by the model are in the next column, E10-E13. These are **transformed** from the values you set.   
     
   This is to keep the parameter values within their physical limits. Remember from the preparation that for the **actual values**:  
     
   "x1 and x3 are positive": this will be true for any values you try in the transformed column, i.e. D10 and D12  
   "x4 is greater than 0.5": this will be true for any value you try in D14  
     
   (x2 can be either positive, zero or negative)

# 1.2 Observations for calibration

We will add the next question from Lecture 2 here:

**h) Do you have observations to calibrate your model?**

1. Yes! We have already seen the observed streamflow data in columns D and E, and the blue line plotted in the main figure (Q).
2. Let's look at these in a little more detail to understand them fully...  
     
   The model actually only uses the **second year of streamflow data** in calibration (i.e. to calculate the "Efficiency criteria" in cells E32-35).   
     
   The first year's data are discarded as part of the model **spin-up** period (in this model usually called the "warm-up" or "warming " period). This is while the model may still be affected by the initial conditions, and is therefore not expected to agree with observations as successfully.  
     
   So the first values of the calibration data being used are actually in D405/E405 (i.e. 1st Jan 1992).  
     
   The **spin-up** and **calibration ("test") time periods** are set in days in cells E20 and E21.
3. This is why the function OFFSET() appears in many equations, because it is skipping down to the second year of data.

# For MAGICC, we skipped the check for calibration data, as we knew there are observations of global mean temperatures. But in general it is important to consider this early on.

# 1.3 Plots of GR4J data

The data are already plotted for you, which is nice.

1. Take a moment to look at the main (Q) and FlowXY figures.  
     
   **Q. Does the model successfully simulate the observations?**   
     
   **Q. When is the model worst? Can you see these data points in the FlowXY figure?**

# 1.4 Summarising the data

For MAGICC, it was natural to use the 2100 value to summarise the temperature projections: for most RCPs, this was the maximum warming.  
  
Now we need to **summarise the streamflow data in one number** that is important to us. Which aspect of streamflow do we most need the model to simulate correctly? For flooding, we care about extremes, so we will use the **maximum streamflow**.

1. We can adapt the calculation for mean observed streamflow, which does the date selection for the calibration period.  
     
   **Copy the formula** (not the cell! - i.e. copy from the equation bar) from cell **E27** to a nearby cell.  
     
   If necessary, increase the number of decimal places of the calculation:  
     
   **Format - Cells... - Number - Number of decimal places: 3**Check this matches the value in E27.
2. Now edit the function from AVERAGE() to **MAX().**  
   **Q. What is the resulting maximum observed streamflow during the calibration period?**  
   Add a text label above or next to this cell, e.g. **Max observed Q (mm/day).**
3. We can now adapt this again for the simulated streamflow.  
    **Copy your new formula (using MAX())** to a cell nearby, and edit the column reference in the formula from BD to AZ.  
     
   **Q. What is the resulting maximum simulated streamflow during the calibration period?**  
     
   Add a text label above or next to this cell, e.g. **Max simulated Q (mm/day).**
4. Now check these values look reasonable by going to the main figure (Q): i.e. whether your results look close to the observed and simulated peak stream flow values during 1992.

2. DECIDE YOUR QUESTION

**Q. Using the state variable of interest and your scenario, write down the scientific question you are investigating:**

3. EVALUATION METRICS

With MAGICC, we performed the evaluation after running the model and downloading data.  
  
With an Excel model, we can calculate the evaluation metric(s) when the model is run, so we will add them to the spreadsheet now. We will calculate both **distance** and **implausibility**.

1. Calculate the **distance** between the maximum values of the simulations and observations. The equation from Lecture 2 for simulation *i* is:   
     
   **di = | yi - yobs |**   
     
   In a cell near the Efficiency Criteria section, calculate the **absolute difference** between the observed and simulated maximum values from the previous section using:  
     
   **=ABS(... - ...)**Add a text label **Distance** next to this value.  
   **Q. What is the distance between the simulated and observed maximum Q?**
2. Calculate the **implausibility**, by dividing the distance by σtot:   
     
   **Ii = | yi - yobs | / σtot**   
     
   where  
     
   **σtot = √( σobs2 + σmod2)**  
     
   In two cells near the distance calculation, enter these values, with labels:  
     
   **Obs error (mm/day)** = 0.1  
   **Model error (mm/day)** = 2xσobs   
     
   I don't have information on the observational error, but the precision of the data is 0.1mm/day, so this seems a reasonable start.  
     
   I have set the model error to be double this, i.e. I don't expect the model error to be smaller than twice the observed error. It is quite a simple model, so we might found this is too small i.e. too strict.
3. Calculate the **total error** using the equation above and the **SQRT()** function, adding a label for this too.
4. Calculate the **implausibility** in a new cell, using the equation above.  
     
   **Q. What values did you calculate for total error and implausibility?  
     
   Q. Would this simulation be ruled out (if requiring I < 3)?**

4. DESIGN YOUR ENSEMBLE

Now design a perturbed parameter ensemble.

# 4.1 Considering design questions

We will consider the following design questions from Lecture 2:

1. Do you have to set up each simulation by hand, or can you automate it?
2. How many runs can you do?
3. How many parameters will you perturb? How will you choose which ones?
4. Do these parameters change simulations of the past?
5. What ranges will you use for each parameter?

**a) Do you have to set up each simulation by hand, or can you automate it?**

**b) How many runs can you do?**

As you know, you change the parameters of GR4J by hand.

Each simulation is extremely fast to setup and run, including changing the parameters, running the model, replotting the data, and calculating the calibration metrics. So we should be able to run more than the 5 simulations we did for MAGICC.

However, we do need to run the past and future scenarios separately, and still need a little time to add each result to our sensitivity analysis. We will aim to start with **a few more than 5**, then potentially add more.

**c) How many parameters will you perturb? How will you choose which ones?**

As the model is much faster than MAGICC to setup and run, we will start by perturbing all **four** parameters.

**d) Do these parameters change simulations of the past?**

Yes - you checked this when you first ran the model.

**e) What ranges will you use for each parameter?**

We will try the **minimum and maximum** possible values that we found in the preparation, to quantify the sensitivity of the predictions to each parameter. This means we can identify the **most influential** parameters - and therefore potentially focus on these in any further simulations.

(This process of narrowing down the number or ranges of parameters being perturbed is sometimes called **screening** or **pre-calibration**.)

The widest ranges for the **actual** parameter values that we found in the preparation were:

**x1:** 0-2500 mm

**x2:** -10 to 5 mm/day

**x3:** 0-1000 mm

**x4:** 0.5-10 days

(The range for x2 was from the wiki, and the other three were from the online model.)

# 4.2 Parameter transformations

Remember that the ranges above are for the **Actual** values of the parameter (shown in cells E10-E13), not the **Transformed** values that you enter in cells D10-D13.

If we want to use the min or max values from above, we must enter them into the **Actual** cells, E10-E13, i.e. **overwriting the equations**.  
  
This is the **quickest and easiest** method. However, by over-writing the equations it does remove the model checks (e.g. that some parameters are positive).

We will have to add our own restrictions, as **the model will not run with the exact lower limits** above for x1, x3 and x4. We have to increase them slightly:

Minimum values:

**x1:** 0.001 mm

**x3:** 0.001 mm

**x4:** 0.501 days

(See the end of the worksheet for an advanced alternative to overwriting the equations).

# 4.3 Perturbed parameter ensemble design

1. Calculate the size of a small factorial ensemble design, using only the default, min and max values for each parameter:  
    **Q. What is the ensemble size for a full factorial design using 3 values for each of the 4 parameters (k =3, *p = 4*)?  
     
   Equation: N = kp**This is too many for one practical session!
2. Calculate the size of a small one-at-a-time ensemble design, using only the min and max values for each parameter:  
    **Q. What is the ensemble size for a OAT design using min/max values for each parameter?  
     
   Equation: N = 1 + 2p**  
   This is much more manageable, and is very close to our aim of 'a few more than 5'.
3. Add a new worksheet to your Excel called **Results**.
4. We will put the **simulations in columns**, not rows, so that it is quick to paste the parameter values into cells E10-E13.  
     
   Add a column at the left that contains the four **parameter names**, and column headers with the **simulation numbers**.
5. Copy and paste the **default parameter values** from E10-E13 into the first column. You need to copy the values, not the formula.  
     
   Either like this:  
   **Copy  
   Edit - Paste Special.... Values**  
   Or paste the values, then select from the clipboard icon that appears:  
   **Values Only**  
     
   It should look like this (and when you click on a cell with a parameter value, it should say the value not the formula):

|  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | **Parameter values** | | |  |  |  |  |  |  |
| **Parameters** | **1** | **2** | **3** | **4** | **5** | **6** | **7** | **8** | **9** |
| **x1** | 320.11 |  |  |  |  |  |  |  |  |
| **x2** | 2.42 |  |  |  |  |  |  |  |  |
| **x3** | 69.63 |  |  |  |  |  |  |  |  |
| **x4** | 1.39 |  |  |  |  |  |  |  |  |

1. Now **finish the one-at-a-time design**, using the min/max values for each parameter from above into each column.  
     
   I find it easiest to paste the min/max values first, like this:

|  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | **Parameter values** | | |  |  |  |  |  |  |
| **Parameters** | **1** | **2** | **3** | **4** | **5** | **6** | **7** | **8** | **9** |
| **x1** | 320.11 | 0.001 | 2500 |  |  |  |  |  |  |
| **x2** | 2.42 |  |  | -10 | 5 |  |  |  |  |
| **x3** | 69.63 |  |  |  |  | 0.001 | 1000 |  |  |
| **x4** | 1.39 |  |  |  |  |  |  | 0.501 | 10 |

1. Then fill out the rest of the table with the default values.

5. RUN THE MODEL: PAST

We will have to run the past and future simulations separately. We will start with the past, so that we can calculate the evaluation metrics at the same time.

# 5.1 Past simulations and evaluation: default run

1. Set up the table rows for your results.  
     
   Below your parameter names, add new row names for the **past and future maximum streamflow** values. I also added a break row with the label Max streamflow:  
     
   **Max streamflow:**  
   **1992  
   Doubled precip**
2. Below this, add new row names for your **evaluation** metrics:  
     
   **Evaluation:**  
   **Distance  
   Implausibility**
3. The model is currently set up to simulate the past using the defaultparameter values, and evaluate this with observations.  
     
   So copy and paste the three relevant results from the model worksheet into the first column:  
     
   **1992:** max simulated Q value   
   **Distance** and **Implausibility:** calculatednear Efficiency criteria   
     
   Remember to **paste the values**, not the formula, as you did before.  
     
   (I moved my implausibility calculation immediately below my distance calculation, so I could copy and paste both at once).

# 5.2 Past simulations and evaluation: rest of the ensemble

1. For each of the other ensemble members:  
     
   a) **Copy the 4 parameter values** from the experimental design table.   
     
   b) **Paste** these into model cells E10-E13  
     
   After you do this the first time, I recommend you delete the values in the **Transformed** column (D10-D12), because these are no longer used by the model.   
     
   c) **Run the model** using the key combination from the start of the worksheet.   
     
   d) **Copy and paste the results** for the 1992 max Q, distance and implausibility into the relevant cells for that simulation.
2. Check your results:  
     
   **Are any values identical between runs?** You might have forgotten to re-run the model before copying the results.  
     
   **Do any results say "#REF!"?** You likely pasted the formula not the value.
3. Now inspect the evaluation metrics.   
     
   **Q. What do you notice about the implausibility values?**

It seems that by using the widest parameter ranges we could find, we used values that were **very unrealistic** for this catchment! (at least, for the weather in 1992).  
  
As we have limited time, **we won't spend time** **running all of these** simulations for the future climate scenario.   
  
(N.B. In Lecture 2, I said DeConto and Pollard (2016) didn't run all their 512 simulations - this is why. They didn't run ensemble members for the future if they failed the calibration).  
  
We can see from the very high implausibility values that **most would be unlikely to pass calibration**, **even if we widened the tolerance interval** a bit (e.g. by increasing σtot). So they would not be included in our calibrated projections.  
  
(If we had more time, we could run them, to see what the uncalibrated projections would be: this can still be interesting/useful).

These results are still very useful, because they tell us:   
  
(a) We should use parameter values **fairly close to the default** values;   
(b) Which parameters **most influence** the past simulations.  
  
So we can try to find more parameter values that would pass calibration, focusing on the most important parameters.

Let us therefore move to the sensitivity analysis...

6. SENSITIVITY ANALYSIS

You will now calculate the **sensitivity** of the model results to each parameter. The method was described in the MAGICC consolidation quiz. You will then visualise the sensitivity for the two most influential parameters.

# 6.1 Quantifying sensitivity

1. Make a table below your results called **Sensitivity Analysis,** with a row for each parameter.
2. For each parameter, calculate the **absolute difference** between the two streamflow values using the min and max of that parameter.  
     
   e.g. for x1, calculate:  
     
   **=ABS( y2 - y3)**where y is the 1992 max Q value, and the index is the run number. **Q. What is the order of parameters by influence, from most to least?**
3. So the **maximum amounts of water in the two stores** strongly affect this model.   
     
   This makes physical sense: the maximum quantity of water that can be released from these two reservoirs will clearly be important for the maximum runoff.

# 6.2 Visualising sensitivity

1. We will **visualise** the sensitivity of the maximum streamflow tothese two parameters, to get an idea of which values might be more plausible than the extremes.  
     
   Insert an **X-Y scatter** chart. Add a data series that plots the values of the parameter **x1** on the x-axis and the results (**1992 max Q**) on the y-axis.
2. Add the equivalent chart for **x3** (or add this as another series to the same figure if you prefer).
3. Label the axes, and add a title (and a legend if plotting on the same figure).
4. Inspect the figure(s) for a while, to understand what is shown.  
    **Q. Where would the observed maximum streamflow be plotted on this figure?**
5. Look at the x1 series.  
     
   **Q. Which data points use the min and max values of x1?**  
   **Q. Which data points in the x1 series shows the default simulation?**
6. Imagine a line joining the three points in the x1 series that you identified above, i.e.:  
     
   Min value of x1 -- Default simulation -- Max value of x1  
     
   **Q. Does this imagined line suggest there is a linear relationship between x1 and maximum streamflow, or nonlinear?**
7. Repeat steps 5 and 6 for x3.  
     
   **Q. Does this imagined line suggest there is a linear relationship between x3 and maximum streamflow, or nonlinear?**

7. RUN YOUR MODEL: PHASE 2

Now that we have **quantified** the sensitivity to the four parameters, and **visualised** the sensitivity to the two most important parameters, we will choose to run some more simulations that should be more successful at reproducing the observations.

This is **Phase 2** of our ensemble design. In history matching studies, these are often called **Waves** of simulations.

# 7.1 Choosing new simulations to run

1. We will add **two runs exploring x3**, the maximum water in the routing store, because this was the most influential parameter. Our aim is to identify more simulations that are consistent with the observations.   
     
   Using the line you imagined for x3, consider **what other values of x3 might also be successful** at simulating the observations, aside from the default value.  
     
   **Q. Roughly what range of x3 values on the x-axis do you estimate would simulate streamflow values around the observed value on the y-axis?**
2. **Add two more columns** to your table: one with x3 = 60 mm and the other with x3 = 80 mm, and default values for the other three parameter values.
3. **Run the model,** and add the results and evaluation, for these two ensemble members.  
     
   **Q. What are the implausibility values?**

# 7.2 Re-evaluating the ensemble

Two of the ensemble members have implausibility I < 3 (runs 1 and 10).

Two others are fairly close to this threshold (runs 8 and 11). Should we accept these?

1. **Looking at the main figure** (Q) for the more implausible simulation (run 11), the streamflow values seem generally good. The peak streamflow in 1992 is a bit small, but not unreasonably so, especially considering the model is very simple. The simulation seems adequate.
2. **Considering our initial judgement for the model error**, we were quite conservative (σmod = 2σobs). If we had used σmod = 3 σobs, all four would have passed. In hindsight, a more generous model error would probably have been more appropriate.   
     
   If we had more time, we could go back and calculate new implausibility values using a larger σtot.
3. Alternatively, it is quicker to change to using a **threshold for the distance values**, instead of I < 3, because the distance values do not need recalculating if we wish to change the threshold. We just adjust the threshold itself.  
     
   We will use a distance threshold of **1mm/day** for calibration: all four ensemble members pass (all have distance < 1 mm/day).   
     
   A more detailed discussion of this is in the **Consolidation and further thinking** quiz.

8. RUN YOUR MODEL: FUTURE

Now we will run a subset of the ensemble for the future climate scenario.

1. We will simulate the future climate scenario (doubled precipitation) with these **four ensemble members**.
2. Copy the **values** of your **doubled precipitation** column from the **Forcing** worksheet into the Precip column (B) of the model.  
     
   **Remember:** while the forcing is set to this future scenario, you must ignore all the evaluation/calibration information in the model!   
     
   i.e. the values of distance and implausibility, and the scatter plot FlowXY, are not relevant because they are comparing with present day observation.  
     
   The dates can also be ignored, of course.
3. For each of the four ensemble members:  
     
   a) **Copy the parameter values** from the design table.   
     
   b) **Paste** these into cells E10-E13  
     
   c) **Run the model**.   
     
   d) **Copy and paste the result** for the simulated max Q into the **Doubled precip** row in the column for that simulation.   
     
   Remember this is still the maximum over just one year: the second year of our future climate simulation.
4. We will skip making a **future vs past** figure, because we have already selected a subset of ensemble members to run the future simulations (and we are short of time).   
     
   But if you had time to run all the ensemble members for the future scenario, this would be a good time to plot this.

9. PRESENT AND INTERPRET

1. Your scientific question might have been something like: "What is the projected maximum streamflow in Le Léguer à Belle-Isle-en-Terre, under a doubling of the precipitation observed in 1992?  
     
   **Q. What is the calibrated range of maximum streamflow in the future climate scenario?**
2. Or else you might have expressed the question like this:  
   "What is the projected **change** in maximum streamflow..."  
     
   We doubled the precipitation in the scenario, so it would be interesting to see if the maximum streamflow also doubles.  
     
   Add a row below your future scenario results labelled **Ratio.**
3. Calculate the ratio of future to present day maximum streamflow for your four ensemble members.
4. **Q. What is the calibrated range of the change in maximum streamflow between the 1992 and future climate scenario?**
5. **Q. Is this a doubling?**

In other words, the relationship between maximum streamflow and precipitation is **nonlinear**: a doubling of rainfall has led to more than double the peak flow. The relationship seems quite consistent across our small calibrated ensemble.  
  
This would be interesting to explore further. Some possible ideas:

a) **Explore more parameter values**, to see if relationship is robust e.g. under parameter interactions (there is more discussion of this in the Consolidation and further thinking quiz(;

b) **Read more** about this model, to understand why, and more about other models and studies, to see if they find the same behaviour;

c) Try **another climate scenario:** e.g. 4 x precipitation, to see if the scaling is the same;

d) Try **different initial values** of the fill fraction of the two stores, to see if changes the results.

FURTHER READING

As an aside, hydrology modellers often use a calibration method that is similar in concept to history matching, i.e. using a threshold rather than tuning. They use the word **equifinality** for the idea that multiple parameter values can be consistent with observations**,** and use the term **behavioural** to for the 'Not Ruled Out Yet' simulations (i.e. that pass calibration). You might come across this in your reading.

I also mentioned briefly in Lecture 2 that one of the chapters in the module textbook uses a history matching approach with a rainfall-runoff model:

Goldstein et al. (2013), [*Assessing Model Adequacy*](https://onlinelibrary.wiley.com/doi/10.1002/9781118351475.ch26). Chapter 26 in: Wainwright and Mulligan, Environmental Modelling: Finding Simplicity in Complexity, John Wiley & Sons, Ltd.

It is written by statisticians that work in the same area of uncertainty quantification as me, i.e. using the same language of history matching and model error. It is quite a tough read! But you might find parts of it useful.

**Advanced:** **Transforming the Parameters**  
  
If you found it a bit unsatisfying to delete the model transformation equations, there are two alternative methods.   
  
First, restore the equations by redownloading the model.

**1. Reverse the transformation**.  
  
If you are fairly confident in maths, you might like to try reversing the transformation, to calculate the corresponding min and max transformed values.   
  
(Hint: you need the formulas **LOG(**..., **2.7182818)** and **ASINH()**. You also need to increase the minimum values a tiny bit to avoid an error in LOG().).  
  
The method and results are shown in the example answer Excel file.

**2. Find the approximate transformed values.**  
  
You could also just try different values of the transformed parameters, to find the min and max values that lead to the above ranges by trial and error.